

## MMAT5010 Linear Analysis (2024-25): Homework 6

Deadline: 15 Mar 2025

### Important Notice:

♣ The answer paper must be submitted before the deadline.

♠ The answer paper MUST BE sent to the CU Blackboard.

1. (a) Let  $\|\cdot\|$  and  $\|\cdot\|'$  be equivalent norm functions on a vector space  $V$ . Show that if  $(V, \|\cdot\|)$  is separable, then so is  $(V, \|\cdot\|')$ .  
(b) Show that every finite dimensional normed space is separable.
2. Assume that  $\mathbb{R}^2$  is endowed with the usual norm, that is  $\|(x_1, x_2)\| := \sqrt{x_1^2 + x_2^2}$  for  $(x_1, x_2) \in \mathbb{R}^2$ . Define  $T : \mathbb{R}^2 \rightarrow (\mathbb{R}^2)^*$  by

$$T(v)(w) := v_1 w_1 + v_2 w_2$$

for  $v = (v_1, v_2)$  and  $w = (w_1, w_2)$  in  $\mathbb{R}^2$ . Show that  $T$  is linear isometric isomorphism.

\*\*\* **End** \*\*\*